

Lesson Plan: Teaching Introductory Calculus (Integration) by using CO₂ Emissions Data

As a **high school** or **undergraduate Mathematics** teacher, you can use this set of computer-based tools to help you in teaching topics such as **integration, definite integral, area under a curve,** and **Riemann sum** in **Introductory Calculus**.

This lesson plan allows students to understand Riemann sum, calculate the area under a curve using Riemann sum, and explore how this value converges to a definite integral. The activity helps students to apply the Riemann sums method for analysis and comparison of data on CO₂ emission, which is considered to be a significant contributor to climate change.

Thus, the use of this lesson plan allows you to integrate the teaching of a climate science topic with a core topic in Mathematics.

Use this lesson plan to help your students find answers to:

- *For a given function $f(x)$ and n , calculate the left Riemann sum and right Riemann sum.*
- *For the same $f(x)$ (as above) and double the value of n (from above), calculate and compare the left and right Riemann sums.*
- *Using Riemann sums, calculate and compare the total CO₂ emissions (data records provided in the activity) for the U.S. and China from 1980 to 2015. What are the possible effects of these CO₂ emissions on the Earth's climate?*

About the Lesson Plan

Grade Level	High school, Undergraduate
Discipline	Mathematics
Topic(s) in Discipline	Calculus, Integration, Definite Integral, Riemann Sum, Area under a Curve

Climate Topic	Energy, Economics, and Climate Change; Climate and the Anthroposphere; Policies, Politics, and Environmental Governance
Location	USA and China
Access	Online, Offline
Language(s)	English
Approximate Time Required	90 – 120 min

1 Contents

- 1. Reading and Associated Activity (30 – 60 min)**

A reading that introduces Riemann sum and the types of Riemann sums. It describes the calculation of the area under a curve by using Riemann sum, and explains how this value can converge to the definite integral.

https://mathinsight.org/calculating_area_under_curve_riemann_sums
- 2. Classroom/Laboratory activity (~60 min)**

A classroom/laboratory activity to analyze CO₂ emissions data for the U.S. and China by using Riemann sums for the calculation of area under the curve.

<http://sustainabilitymath.org/calculus-materials/>
- 3. Suggested questions/assignments for learning evaluation**

 - For a given function $f(x)$ and n , calculate the left Riemann sum and right Riemann sum.
 - For the same $f(x)$ (as above) and double the value of n (from above), calculate and compare the left and right Riemann sums.

- Using Riemann sums, calculate and compare the total CO₂ emissions (data records provided in the activity) for the U.S. and China from 1980 to 2015. What are the possible effects of these CO₂ emissions on the Earth's climate?

2 Step-by-step User Guide



Here is a step-by-step guide to using this lesson plan in the classroom/laboratory. We have suggested these steps as a possible plan of action. You may customize the lesson plan according to your preferences and requirements.

1. Introduce the topic by using a reading and an associated activity

- Introduce the topic of Riemann sum and the types of Riemann sums by using the reading, "[Calculating the area under a curve using Riemann sums](#)" from D. Q. Nykamp at Math Insight.
- Conduct the activities using the applets to further explain how to calculate the area under a curve by using the Riemann sum and how this value can converge to the definite integral.

The reading "Calculating the area under a curve using Riemann sums" and the associated activities (using applets) are available at https://mathinsight.org/calculating_area_under_curve_riemann_sums.

2. Conduct a classroom/laboratory activity

Then, help your students apply the learned concepts through a hands-on classroom/laboratory activity, "[U.S. and China CO₂ Emissions](#)", by Thomas J. Pfaff at Sustainability Math. This activity uses CO₂ emission data and population data for the U.S. and China for the period 1980 to 2015.

This activity will help students to

- analyze CO₂ emissions for each country
- compare the CO₂ emissions for the countries by using Riemann sums for the data from 1980 to 2015

- create a proposal for emission reduction by considering past and current CO₂ emissions for the two countries

Go to <http://sustainabilitymath.org/calculus-materials/>.

- Download the material in the project, “U.S. and China CO₂ Emissions”, under Calculus I – Integration Related Projects.
- Students can perform the exercises described in the Word file by using the data in the Excel file.

3. Questions/Assignments

Use the tools and the concepts learned so far to discuss and determine answers to the following questions:

- *For a given function $f(x)$ and n , calculate the left Riemann sum and right Riemann sum.*
- *For the same $f(x)$ (as above) and double the value of n (from above), calculate and compare the left and right Riemann sums.*
- *Using Riemann sums, calculate and compare the total CO₂ emissions (data records provided in the activity) for the U.S. and China from 1980 to 2015. What are the possible effects of these CO₂ emissions on the Earth’s climate?*

3 Learning Outcomes

The tools in this lesson plan will enable students to:

- calculate the approximate area under a curve by using the Riemann sums method
- compare the results obtained for left and right Riemann sums for the same curve
- explain how the estimate of the area under a curve (using Riemann sum) converges to the definite integral
- apply the Riemann sum method to analyze and compare CO₂ emissions data for the U.S. and China

4 Additional Resources



If you or your students would like to explore the topic further, these additional resources will be useful.

1. Videos

Video tutorial, “Riemann sums”, from Khan Academy:

<https://www.khanacademy.org/math/old-integral-calculus/riemann-sums-ic>

5 Credits/Copyrights

All the teaching tools in our collated list are owned by the corresponding creators/authors/organizations as listed on their websites. Please view the individual copyright and ownership details for each tool by following the individual links provided.

We have selected and analyzed the tools that align with the overall objective of our project and have provided the corresponding links. We do not claim ownership of or responsibility/liability for any of the listed tools.

1. **Reading and Associated Activity, “Calculating the area under a curve using Riemann sums”** D. Q. Nykamp; from [Math Insight](#)
2. **Classroom/Laboratory Activity, “U.S. and China CO2 Emissions”** Thomas J. Pfaff, [Sustainability Math](#)
3. **Additional Resources** [Khan Academy](#)