### Lesson Plan: Teaching Differential Calculus (Application of Derivatives) using Arctic Sea Ice Data

As a **high school** or **undergraduate Mathematics** teacher, you can use this set of computer-based tools to help you in teaching introductory **differential calculus** and specifically **the application of derivatives**.

This lesson plan will allow you to teach introductory derivatives, polynomial differentiation, and the application of derivatives. The lesson plan includes a hands-on computer-based classroom activity to be conducted on datasets of Arctic Ice Data (1980-2017). This activity includes a set of inquiry-based questions that will enable your students to apply their understanding of sixth degree polynomial differentiation, maxima/minima values, finding roots and inflection points.

Thus, the use of this lesson plan allows you to integrate the teaching of a climate science topic with a core topic in **Mathematics**.

Use this lesson plan to help your students find answers to:

- What are derivatives and their functions?
- Using an example, describe polynomial differentiation.
- Is the extent of the Arctic Sea Ice decreasing since 1980?
- Has the speed of melting of Arctic Sea Ice changed from 1980- 2017?
- Discuss the Ice Albedo Feedback and Global Warming to explain the differences in rates of melting of and extent of Arctic Sea Ice over the past four decades.

**About the Lesson Plan** 

Grade Level: High School, Undergraduate **Discipline:** Mathematics Topic(s) in Discipline: Derivatives, Polynomial Differentiation, Function Graph, Extrema, Concavity, Roots, Inflection Points Climate Topic: Climate and the Cryosphere, Climate Variability Record **Location:** Global Access: Online, Offline Language(s): English **Approximate Time Required: 60-75 min** 

### 1 Contents

### 1. Teaching Module (30 min)

A teaching module to explain the basics of derivatives, derivative rules, and differentiation.

This can be accessed at:

https://www.khanacademy.org/math/ap-calculus-bc/bc-differentiation-1-new

### 2. Teaching Module (20 min)

A teaching module to explain the application of derivatives in comparing rates in differentiation and aspects of the function graph.

This can be accessed at:

https://openstax.org/books/calculus-volume-1/pages/4-5-derivatives-and-the-shape-of-a-graph

### 3. Classroom Activity (20 min)

A classroom activity to apply understanding of differential rates using datasets of Arctic Sea Ice Data (1980-2017).

This can be accessed at:

http://sustainabilitymath.org/calculus-materials/

A pre-activity reading can be accessed at:

### http://homes.sice.indiana.edu/donbyrd/Teach/Math/Polynomials+Derivatives.pdf

### 4. Visualization (5 min)

An interactive visualization of changes in the extent of Arctic Sea Ice from 1979-2020.

This can be accessed at:

https://nsidc.org/arcticseaicenews/charctic-interactive-sea-ice-graph/

### 5. Suggested questions/assignments for learning evaluation

- What are derivatives and their functions?
- Using an example, describe polynomial differentiation.
- Is the extent of the Arctic Sea Ice decreasing since 1980?
- Has the speed of melting of Arctic Sea Ice changed from 1980- 2017?
- Discuss the Ice Albedo Feedback and Global Warming to explain the differences in rates of melting of and extent of Arctic Sea Ice over the past four decades.

## 2 Step-by-step User Guide

Here is a step-by-step guide to using this lesson plan in the classroom/laboratory. We have suggested these steps as a possible plan of action. You may customize the lesson plan according to your preferences and requirements.

#### 1. Topic introduction and discussion

Use the teaching module, '<u>Differentiation: definition and basic derivative rules</u>' by Khan Academy, to introduce the concept of derivatives and differentiation. Navigate to the sub-sections within the module to explain derivative rules, the power rule, and how to differentiate polynomials. Use the in-built practice exercises and quizzes to evaluate your students' understanding of the topics.

This can be accessed at:

https://www.khanacademy.org/math/ap-calculus-bc/bc-differentiation-1-new

### 2. Develop the topic further

Use the teaching module, '<u>Derivatives and the shape of a graph</u>' by OpenStax<sup>™</sup>, Rice University, to explain to your students how related rates can be compared using derivatives. Use the sub-sections within the tool to explain various mathematical concepts for determining maxima/minima values, roots, and inflection points in the derivative function graph.

This can be accessed at:

https://openstax.org/books/calculus-volume-1/pages/4-5-derivatives-and-the-shape-of-a-graph

### 3. Extend understanding

Use the reading, 'Polynomials and their Derivatives: Polynomials, Critical Points, and Inflection Points' by Donald Byrd, Indiana University Informatics to reiterate the mathematical concepts to be applied in the ensuing activity.

Use the classroom activity, 'Arctic Sea Ice' from Sustainability Math by Thomas J. Pfaff, Professor of Mathematics, Ithaca College, USA, to enable your students to apply their understanding of derivatives, polynomial differentiation and application of derivatives using datasets from the National Snow and Ice Data Center (NSIDC). This classroom activity includes three datasets of the extent of Arctic Sea Ice linked from NSIDC's observations from 1980 to 2017. This data is provided in an Excel spreadsheet. The classroom activity also includes a Word document that contains directions on how to use different mathematical methods on the data provided. It further includes questions that you may wish to use in your classroom to explain mathematical functions and methods and to initiate a discussion on the decrease in extent of Arctic Sea Ice due to the Ice Albedo Feedback and anthropogenically forced Global Warming (links to explanatory notes given within the tool). Direct your students to download the Excel file (with dataset) and the Word document (with directions to use the dataset and a set of questions to analyze the dataset). Proceed with the classroom activity and encourage your students to answer the questions by applying their understanding of function graph, maxima/minima, roots, and inflection points.

This can be accessed at:

http://sustainabilitymath.org/calculus-materials/#

#### 4. Discuss further

Use the visualization, 'Charctic Interactive Sea Ice Graph' from NSIDC to encourage discussion amongst your students about the changes in the extent of Arctic Sea Ice from the years 1979-2020. Discuss how these changes could be the result of changes in the Earth's climate in recent times.

These can be accessed at:

https://nsidc.org/arcticseaicenews/charctic-interactive-sea-ice-graph/

### 5. Questions/Assignments

Use the tools and the concepts learned so far to discuss and determine answers to the following questions:

- What are derivatives and their functions?
- Using an example, describe polynomial differentiation.
- Is the extent of the Arctic Sea Ice decreasing since 1980?
- Has the speed of melting of Arctic Sea Ice changed from 1980- 2017?
- Discuss the Ice Albedo Feedback and Global Warming to explain the differences in rates of melting of and extent of Arctic Sea Ice over the past four decades.

### 3 Learning Outcomes

The tools in this lesson plan will enable students to:

- learn about derivatives and their application in comparing rates
- understand the derivative function graph and related terms such as extrema, roots, and inflection points
- use function graphs to observe changes in rates of melting and the extent of Arctic Sea Ice in given datasets

# 4 Credits/Copyrights

All the teaching tools in our collated list are owned by the corresponding creators/authors/organizations as listed on their websites. Please view the individual copyright and

ownership details for each tool by following the individual links provided.

We have selected and analyzed the tools that align with the overall objective of our project and have provided the corresponding links. We do not claim ownership of or responsibility/liability for any of the listed tools.

1. Teaching Module; 'Differentiation: definition and basic derivative rules'

Developed by Khan Academy

2. Teaching Module; 'Derivatives and the Shape of a Graph'

Provided by <u>OpenStax™</u>, Rice University.

3. Classroom Activity; 'Arctic Sea Ice'

Provided by Sustainability Math by Thomas J. Pfaff, Professor of Mathematics, Ithaca College, USA.

4. Reading; 'Polynomials and their Derivatives'

By **Donald Byrd**, Indiana University Informatics.

5. Visualization; 'Charctic Interactive Sea Ice Graph'

From National Snow and Ice Data Center (NSIDC)